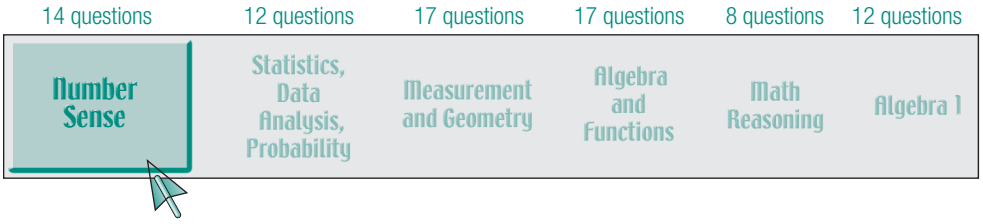
Number Sense Strand



Fourteen of the 80 CAHSEE math questions are based on ten selected standards from the seventh grade Number Sense strand.

WHAT DO THE NUMBER SENSE STANDARDS ASK ME TO DO?

The CAHSEE Number Sense strand includes basic arithmetic calculations with whole numbers, fractions, and decimals, all done without using a calculator.

The CAHSEE Number Sense questions will ask you to:

- solve problems with fractions, decimals, percentages, and integers
- compare and order numbers
- understand percents, including percents less than 1% and greater than 100%
- use ratios and proportions to solve problems
- understand the meaning of numbers written in scientific notation
- find and use multiples, factors, and primes
- add, subtract, multiply, and divide numbers, and use the relationships among these operations including inverse, commutative, associative, and distributive properties
- estimate square roots of whole numbers to the nearest whole number

Vocabulary

The following words have appeared on the CAHSEE previously. If any of these words are unfamiliar to you, look them up in the CAHSEE Math Vocabulary list in the appendix at the back of this study guide, or check with your math teacher.

absolute value	integer	scientific notation
decreased by	prime	square
equivalent expression	simple interest	square root

WHY IS NUMBER SENSE IMPORTANT?

As an adult, you will use all the basic skills of arithmetic included in the Number Sense strand. You'll use your number sense skills as a consumer (nutrition choices, shopping for credit, choosing the best buy), as a U.S. citizen (taxes, voting issues), and as an employee (handling money, cost estimates, profit and loss, quality control). Many large companies give screening tests that include number-sense knowledge for entry-level jobs.

The anchor problem for this strand, *Emergency Refrigerator!,* presents a situation that you may encounter when you rent your first apartment. But before we try *Emergency Refrigerator!,* let's first look at some released CAHSEE questions, with solutions, that apply to this strand.

HOW WILL THE CAHSEE TEST MY KNOWLEDGE OF NUMBER SENSE?

The CAHSEE tests ten of the 12 standards from the Number Sense strand for seventh grade. Let's start by looking at eight of these standards and some actual CAHSEE questions based on them. Each box that follows contains one of the standards, a released question based on that standard, and a solution with explanation.

NS 1.1 Read, write, and compare rational numbers in scientific notation (positive and negative powers of 10) with approximate numbers using scientific notation. [1 question]

Released CAHSEE Question

The radius of the earth's orbit is 150,000,000,000 meters. What is this number in scientific notation?

A 1.5×10^{-11}

B 1.5×10^{11}

C 15×10^{10}

D 150×10^{9}

M00213

Solution

Scientific notation is a short way to write very large or very small numbers using powers of 10. Here are some examples of the same numbers written first in the usual way and then in scientific notation:

$2000 \rightarrow 2 \times 10^{3}$ $\quad 143000 \rightarrow 1.43 \times 10^{5}$ $\quad 0.0000234 \rightarrow 2.34 \times 10^{-5}$

A number in scientific notation is always written as a number greater than or equal to 1 but less than 10, times a power of 10. Looking at the possible answers for this CAHSEE question, you can see that options C and D are both incorrect because 15 and 150 are larger than 10. Also, in option A, the 10 has a negative exponent, so it would be a very small number. The correct answer must be B. But why?

You can rewrite 150,000,000,000 as $15 \times 10{,}000{,}000{,}000$, which is 15×10^{10}. But "15×10^{10}" is not yet in scientific notation because "15" is not a number between 1 and 10. So think of 15 as 1.5×10^{1}. Then $15 \times 10^{10} = (1.5 \times 10^{1}) \times 10^{10} = 1.5 \times 10^{11}$, choice **B**.

NS 1.2 Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers. [3 questions]

Released CAHSEE Question

$\frac{11}{12} - \left(\frac{1}{3} + \frac{1}{4}\right)$

A $\frac{1}{3}$

B $\frac{3}{4}$

C $\frac{5}{6}$

D $\frac{9}{5}$

M02048

Solution

One way to do this problem is to first find the least common denominator for the three fractions. Notice that the numbers 3, 4, and 12 all divide evenly into 12, so 12 is the least common denominator. Next, find equivalent fractions for $\frac{1}{3}$ and for $\frac{1}{4}$, each with 12 as the denominator:

$$\frac{1}{3} = \frac{1}{3} \bullet \frac{4}{4} = \frac{4}{12} \text{ and } \frac{1}{4} = \frac{1}{4} \bullet \frac{3}{3} = \frac{3}{12}$$

Finally, the numerators of these fractions can be combined to get the solution. You could write your work out like this:

$$\frac{11}{12} - \left(\frac{1}{3} + \frac{1}{4}\right) = \frac{11}{12} - \left(\frac{4}{12} + \frac{3}{12}\right) = \frac{11}{12} - \frac{7}{12} = \frac{4}{12}$$

But $\frac{4}{12}$ isn't an answer choice! You need to reduce $\frac{4}{12}$ to get $\frac{1}{3}$, so the correct answer is **A**.

NS 1.6 Calculate the percentage of increases and decreases of a quantity. [1 question]

Released CAHSEE Question

The price of a calculator has decreased from $12.00 to $9.00. What is the percent of decrease?

A 3%

B 25%

C 33%

D 75%

M02868

Solution

A price change from $12 down to $9 is a net decrease of $3. To find the percent decrease (or percent increase), the base is always the original or starting number, in this case $12. So, the correct percent decrease is $3/12 = 25\%$, choice **B**. Notice that $3/9 = 33\%$, option C, is not correct because $9 is the ending price, not the starting price.

NS 2.1 Understand negative whole-number exponents. Multiply and divide expressions involving exponents with a common base. [1 question]

Released CAHSEE Question

$$\frac{10^{-2}}{10^{-4}} =$$

A 10^{-6}

B 10^{-2}

C 10^{2}

D 10^{8}

M02832

Solution

When calculating with numbers written in scientific notation, it's important to know how to multiply and divide powers of ten.

Here are a few powers of 10 and their equivalents written in the usual way:

$$10^4 = 10{,}000$$
$$10^3 = 1{,}000$$
$$10^2 = 100$$
$$10^1 = 10$$
$$10^0 = 1$$
$$10^{-1} = 0.1 = \frac{1}{10}$$
$$10^{-2} = 0.01 = \frac{1}{100}$$
$$10^{-3} = 0.001 = \frac{1}{1000}$$

To simplify $\frac{10^{-2}}{10^{-4}}$, one way is to rewrite

$$\frac{10^{-2}}{10^{-4}} = \frac{\frac{1}{100}}{\frac{1}{10{,}000}} = \frac{1}{100} \div \frac{1}{10{,}000} = \frac{1}{100} \times \frac{10{,}000}{1} = 100 = 10^2$$

Therefore, the correct answer is 10^2, choice **C**.

Another way is to remember that a negative power of ten is just the reciprocal of the positive power of ten. Using this idea, $10^{-2} = \frac{1}{10^2}$ and $\frac{1}{10^{-4}} = 10^4$, therefore:

$$\frac{10^{-2}}{10^{-4}} = \frac{10^4}{10^2} = 10^2.$$

A third way is to remember the "law of exponents" for dividing powers of the same base:

$\frac{a^m}{a^n} = a^{(m-n)}$. So for this problem, $\frac{10^{-2}}{10^{-4}} = 10^{(-2-(-4))} = 10^{(-2+4)} = 10^2$.

NS 2.2 Add and subtract fractions by using factoring to find common denominators. [1 question]

Released CAHSEE Question

Which of the following is the prime factored form of the lowest common denominator of $\frac{7}{10} + \frac{8}{15}$?

A 5×1

B $2 \times 3 \times 5$

C $2 \times 5 \times 3 \times 5$

D 10×15

M02826

Solution

The denominators of these two fractions are 10 and 15. In order to add or subtract these fractions you would need to find a common denominator (a number that both 10 and 15 divide into evenly). One way to do this is to list the multiples of the larger number, 15, until you get a multiple that the smaller number also divides into evenly. Multiples of 15 are 15, 30, 45, 60, and so on. The lowest number in this list that 10 also divides evenly is 30. Therefore, 30 is the least common denominator. So, which of the multiple choice answers multiplies out to 30? Only choice **B**, which is the correct answer.

But notice that standard NS 2.2 says that you are to "use factoring to find the common denominator." According to the California standards, the proper way to do this problem is to first find the prime factorization of each denominator: $10 = 2 \times 5$ and $15 = 3 \times 5$. Then the common denominator is the product of the smallest set of prime factors that are common to both prime factorings, in this case $2 \times 3 \times 5$.

Notice that option C is incorrect. Even though $2 \times 5 \times 3 \times 5 = 150$, which is a common denominator, it is not the *least* common denominator; the factor 5 doesn't need to be included twice.

Sometimes the method specified in the California standards is easier to use, particularly if the fractions' denominators are large numbers or have many factors.

NS 2.3 Multiply, divide, and simplify rational numbers by using exponent rules. [1 question]

Released CAHSEE Question

$(3^8)^2 =$

A 3^4

B 3^6

C 3^{10}

D 3^{16}

M02406

Solution

Sometimes it's difficult to remember how to use exponents. But you can answer these questions correctly if you understand how to use them. The exponent tells you how many times the base number is multiplied by itself. So,

$$(3^8)^2 = (3^8)(3^8) = (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3) = 3^{16}.$$

Choice **D** is correct.

NS 2.4 Use the inverse relationship between raising to a power and extracting the root of a perfect square integer; for an integer that is not square, determine without a calculator the two integers between which its square root lies and explain why. [1 question]

Released CAHSEE Question

The square root of 150 is between

A 10 and 11

B 11 and 12

C 12 and 13

D 13 and 14

M02666

Solution

The square root of a number is a number that can be multiplied by itself to get the original number. Some numbers have square roots that are integers, for example 49. The square root of 49 is 7, because $7 \times 7 = 49$. But most numbers, like 150, do not have square roots that are integers. So, instead of figuring out the square root of 150, let's look at the squares of the answer choices for this question:

$10 \times 10 = 100$; $11 \times 11 = 121$; $12 \times 12 = 144$; $13 \times 13 = 169$; $14 \times 14 = 196$.

Because 150 is between 144 and 169, the square root of 150 must be between 12 and 13. The correct answer is **C**.

NS 2.5 Understand the meaning of the absolute value of a number; interpret the absolute value as the distance of the number from zero on a number line; and determine the absolute value of real numbers. [1 question]

Released CAHSEE Question

If $|x| = 3$, what is the value of x?

A -3 or 0

B -3 or 3

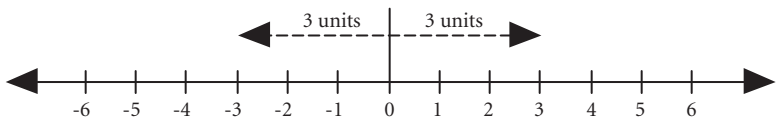
C 0 or 3

D -9 or 9

M02122

Solution

The absolute value of a number is its distance away from 0 on the number line. So if $|x| = 3$, then x must be 3 units away from 0.



The number line above shows there are two such numbers, 3 and -3, so the correct answer is **B**.

USING NUMBER SENSE STANDARDS IN A REAL-LIFE SITUATION

You might be asking yourself, "When will I ever need to use this stuff?" To help you get the "big picture," the remaining two Number Sense standards are illustrated by a real-life problem *Emergency Refrigerator!* that a person might encounter after high school. Even though the CAHSEE doesn't include problems like this one, you might find it easier to remember one large problem—an "anchor problem"—in which many of the skills are combined, rather than trying to recall each of the standards individually.

Try to do this problem before you look at its solution on the following pages.

Emergency Refrigerator!

You're just getting comfortable living in your first apartment. Then—emergency! The old refrigerator left to you by the previous tenants stops working. You need to get a new refrigerator immediately. You decide on the brand and model of refrigerator you want and you have the following three options for purchasing it:

Option #1
Wagmen's Department Store has the refrigerator for $240, but this week only it is marked as part of their "red sticker sale." The red sticker means it will be sold at "1/4 off" the list price. Delivery is $20.

Option #2
The same refrigerator is advertised at Big Box Discount Appliances for $210. But you are lucky; this weekend only they have a coupon in the paper for 15% off the total cost of any item with a regular price over $100. Delivery costs $30.

Option #3
A friend works for Mike's Furniture and can get the same refrigerator for you for 20% over the wholesale price of $180. Your friend can use the company truck to deliver it for free.

You'll need to figure out the cost of each option before deciding what to do. Try to work out the cost of each option before going on to the next page. Remember, no calculators are allowed!

Emergency Refrigerator! Solution and Standards

Are you ready to check your answers to the *Emergency Refrigerator!* anchor problem? In order to figure the cost of the options, you need to use two of the Number Sense standards that are tested on the CAHSEE. The standards that apply in this situation, along with the number of questions on the CAHSEE that are based on that standard, appear to the right in the small print.

To decide which option has the lowest price, you'll need to calculate the cost of each option.

Option #1

One-fourth of $240 is $60, the amount to be taken off. The sale price would be $240 − $60 = $180. But to get the refrigerator, you'll have to get it delivered for $20. So the total cost for Option #1 would be **$200**.

NS 1.3 Convert fractions to decimals and percents and use these representations in estimations, computations, and applications. [2 questions]

Option #2

Because the refrigerator costs more than $100, you can use the 15% off coupon to discount the selling price by 15%. One method is to first find 15% of $210. One way to think of 15% is "15 cents per dollar." So, 15% of 210 is 0.15 × $210, which is $31.50. Then subtract $31.50 from the advertised price of $210, giving a sale price of $178.50.

Another way to figure the sale price is to realize that if you get 15% off, that means you need to pay only 85% of the advertised price. 85% of $210 is $178.50.

Finally, you need to add the $30 delivery charge, giving a total cost of **$208.50**.

NS 1.7 Solve problems that involve discounts, markups, commissions, and profit and compute simple and compound interest. [2 questions]

Option #3

In order to figure the cost of this option, you need to find your price by adding on a 20% markup to the wholesale price of $180. Markups can be done in two ways. One way is to find 20% of $180, which is $36, and then add the $36 to the $180 to get the selling price of $216. Another way is to realize that if 20% is to be added on, then the selling price is 120% of the wholesale price. Then 120% of 180 is **$216**. Either way you do it, that's the total cost for Option #3 because there is no delivery charge.

You know that Option #1 costs $200, Option #2 costs $208.50, and Option #3 costs $216. Now that you've done the math, which option would you choose?

You've seen the ten Number Sense standards; now you are ready for a practice test. Answer the questions in the next section—the Practice Test—and then check your answers using the answer key provided at the end.

(Note: The CAHSEE questions used as examples throughout this Study Guide are questions that were used on prior CAHSEEs. These items will not be used in future CAHSEEs.)

NUMBER SENSE PRACTICE TEST

1. $3.6 \times 10^2 =$

A 3.600
B 36
C 360
D 3,600

M00036

2. **The five members of a band are getting new outfits. Shirts cost $12 each, pants cost $29 each, and boots cost $49 a pair. What is the total cost of the new outfits for all of the members?**

A $90
B $95
C $450
D $500

M00331

3. **If Freya makes 4 of her 5 free throws in a basketball game, what is her free throw shooting percentage?**

A 20%
B 40%
C 80%
D 90%

M00223

4. **The cost of an afternoon movie ticket last year was $4.00. This year an afternoon movie ticket costs $5.00. What is the percent increase of the ticket from last year to this year?**

A 10%
B 20%
C 25%
D 40%

M02158

5. **Sally puts $200.00 in a bank account. Each year the account earns 8% simple interest. How much interest will be earned in three years?**

A $16.00
B $24.00
C $48.00
D $160.00

M02119

6. $4^3 \times 4^2 =$

A 4^5
B 4^6
C 16^5
D 16^6

M02661

7. The square of a whole number is between 1,500 and 1,600. The number must be between

A 30 and 35.

B 35 and 40.

C 40 and 45.

D 45 and 50.

M00313

8. A CD player regularly sells for $80. It is on sale for 20% off. What is the sale price of the CD player?

A $16

B $60

C $64

D $96

M02425

9. What is the absolute value of −4?

A -4

B $-\frac{1}{4}$

C $\frac{1}{4}$

D 4

M02667

NUMBER SENSE PRACTICE TEST ANSWER KEY

Question Number	Standard	Correct Answer
1	NS 1.1	C
2	NS 1.2	C
3	NS 1.3	C
4	NS 1.6	C
5	NS 1.7	C
6	NS 2.3	A
7	NS 2.4	B
8	NS 1.7	C
9	NS 2.5	D